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$$y = \left(\frac{x-5}{2x+1} \right)^3$$

$$\frac{dy}{dx} = y' = 3 \left(\frac{x-5}{2x+1} \right)^2 \frac{d}{dx} \left(\frac{x-5}{2x+1} \right)$$

$$= 3 \left(\frac{x-5}{2x+1} \right)^2 \left(\frac{(1)(2x+1) - (x-5)(2)}{(2x+1)^2} \right)$$

$$= 3 \left(\frac{x-5}{2x+1} \right)^2 \left(\frac{2x+1 - (2x-10)}{(2x+1)^2} \right)$$

$$= 3 \left(\frac{x-5}{2x+1} \right)^2 \left(\frac{11}{(2x+1)^2} \right) = \frac{33(x-5)^2}{(2x+1)^4}$$

$$3.5/39 \quad y = \frac{(2x+3)^3}{(4x^2-1)^8}$$

$$\frac{d}{dx} \left(\frac{(2x+3)^3}{(4x^2-1)^8} \right) = \frac{\frac{d}{dx}(2x+3)^3 (4x^2-1)^8 - (2x+3)^3 \frac{d}{dx}(4x^2-1)^8}{\left((4x^2-1)^8 \right)^2}$$

$$= 3(2x+3)^2 \frac{d}{dx}(2x+3)$$

$$\frac{d}{dx}(4x^2-1)$$

$$= \frac{(3(2x+3)^2)(2) \cdot (4x^2-1)^8 - (2x+3)^3 (8(4x^2-1)^7)(8x)}{(4x^2-1)^{16}}$$

$$\frac{d}{dx}(4x^2-1)^8 = 8(4x^2-1)^7 \frac{d}{dx}(4x^2-1)$$

$$= \frac{(2x+3)^2 (4x^2-1)^7 [6(4x^2-1) - (2x+3)(8)(8x)]}{(4x^2-1)^{16}}$$

$$= \frac{(2x+3)^2 (4x^2-1)^7 [24x^2 - 6 - (128x^2 + 192x)]}{(4x^2-1)^{16}}$$

$$= \frac{(2x+3)^2}{(4x^2-1)^9} (-104x^2 - 192x - 6)$$

3.5/35

$$y = (5x+8)^{13}(x^3+7x)^{12}$$

$$y' = \frac{d}{dx} (5x+8)^{13} (x^3+7x)^{12} + (5x+8)^{13} \frac{d}{dx} ((x^3+7x)^{12})$$

$$= \left(13(5x+8)^{12} \frac{d}{dx} (5x+8) \right) (x^3+7x)^{12} + (5x+8)^{13} \left(12(x^3+7x)^{11} \frac{d}{dx} (x^3+7x) \right)$$

$$= \left(13(5x+8)^{12} (5) \right) (x^3+7x)^{12} + (5x+8)^{13} \left(12(x^3+7x)^{11} (3x^2+7) \right)$$

3.6 Implicit Differentiation

$$f(x) = (5x+8)^{13} (x^3+7x)^{12}$$

$$f(x) = (5x+8)^{13} y$$

$$f'(x) = \frac{d}{dx}((5x+8)^{13}) \cdot y + (5x+8)^{13} \frac{d}{dx}(y)$$

$$= [13(5x+8)^{12}(5)] \cdot y + (5x+8)^{13} \frac{dy}{dx}$$

$$xy + y + 1 = x$$

$$\left[\frac{d}{dx}(x) \cdot y + x \left(\frac{d}{dx}(y) \right) \right] + \frac{d}{dx}(y) = 1$$

$$y + x \frac{dy}{dx} + \frac{dy}{dx} = 1$$

$$x \frac{dy}{dx} + \frac{dy}{dx} = 1 - y$$

$$(x+1) \frac{dy}{dx} = 1 - y$$

$$\frac{dy}{dx} = \frac{1-y}{x+1}$$

$$xy + y = x - 1$$

$$y(x+1) = x - 1$$

$$y = \frac{x-1}{x+1}$$

$$\frac{dy}{dx} = \frac{(x+1) - (x-1)}{(x+1)^2}$$

$$= \frac{2}{(x+1)^2}$$

$$\frac{\left[1 - \left(\frac{x-1}{x+1} \right) \right] (x+1)}{[x+1] (x+1)} =$$

$$\frac{(x+1) - (x-1)}{(x+1)^2} = \frac{2}{(x+1)^2}$$

Function defined explicitly

you
know

$$y = \text{blah} - x - \text{blah} - \text{blah} - x - x$$

Function defined implicitly

$$x^2 + y^2 = 1$$

$$y^2 = 1 - x^2 \begin{cases} \text{either } y = \sqrt{1 - x^2} \\ \text{or } y = -\sqrt{1 - x^2} \end{cases}$$

$$x^2 + y^2 = 1$$

$$2x + 2y \left(\frac{dy}{dx} \right) = 0$$

$$\frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}$$

$$\frac{2y \frac{dy}{dx}}{2y} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

differentiate
implicitly

$$x^3 + y^3 = 3xy \quad (3x)(1y)$$

$$3x^2 + 3y^2 \frac{dy}{dx} = 3 \quad \begin{matrix} f(x) = 3x & f'(x) = 3 \\ g(x) = y & g'(x) = \frac{dy}{dx} \end{matrix}$$

$$3x^2 + 3y^2 \frac{dy}{dx} = (3x \cdot \frac{dy}{dx}) + (y \cdot 3)$$

$$3y^2 \frac{dy}{dx} - (3x \cdot \frac{dy}{dx}) = 3y - 3x^2$$

$$\frac{dy}{dx} (3y^2 - 3x) = 3y - 3x^2$$

$$\boxed{\frac{dy}{dx} = \frac{3y - 3x^2}{3y^2 - 3x}}$$